Coarse homotopy theory and K-theory

U. Bunke
joint with A. Engel (Greifswald)

September 30, 2022

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fix a countable group G and a C^*-category $\mathbf{C}$ with G-action, sufficiently additive

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There is a construction of an equivariant coarse K-homology functor

$$\boxed{\text{category } G\mathbf{BC} \text{ of } G\text{-bornological coarse spaces}} \xrightarrow{K\mathcal{X}^G_{\mathbf{C}}} \boxed{\text{category } \mathbf{Sp} \text{ of spectra}}.$$

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the induced equivariant homology theory is equivariant K-homology:

$$K\mathcal{X}^G_{\mathbf{C}}((G/H)_{\min,\max} \otimes G_{\mathrm{can},\min}) \simeq K(\mathbf{C} \rtimes_r H)$$

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Theorem (B-Engel)

If $E : \mathbf{BC} \to \mathbf{Sp}$ is a strong coarse homology theory and $X \in \mathbf{BC}$ has finite asymptotic dimension, then the coarse assembly map

$$\mu_{E,X} : E\mathcal{O}^{\infty}\mathbf{P}(X) \to \Sigma E(X)$$

is an equivalence.

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for K fin. dim. simplicial complex: $E\mathcal{O}^{\infty}(K) \simeq \Sigma H^{\mathrm{lf}}(K, E(*))$ (provided E is additive)

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$E\mathcal{O}^\infty\mathbf{P}$ - its coarsification
$E\mathcal{O}^\infty\mathbf{P}(X) := \mathrm{colim}_{U\in\mathcal{C}}\, E\mathcal{O}^\infty(\mathrm{Rips}_U(X))$
note: X has bounded geometry iff $\dim(\mathrm{Rips}_U(X)) < \infty$ for all U in $\mathcal{C}$

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Note: CBC is known under much weaker assumptions on X: e.g. coarse embedding into a Hilbert space (G. Yu)

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consider equivariant homology theory:

$$E^G : G\mathbf{Orb} \xrightarrow{S \mapsto S_{\mathrm{min},\mathrm{max}} \otimes G_{\mathrm{can},\mathrm{min}}} G\mathbf{BC} \xrightarrow{E} \mathbf{Sp}$$

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Elmendorf's theorem: $\boxed{\text{$\mathbf{Sp}$-valued equiv. homology theories}} \overset{H^G \mapsto (S \mapsto H^G(S))}{\simeq} \mathbf{Fun}(G\mathbf{Orb}, \mathbf{Sp})$

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Theorem (B-Engel-Kasprowski-Winges)

*If E is continuous, strongly additive and admits transfers, G_{can} has **Fin**-finite decomposition complexity, and there exists a finite-dimensional CW-complex $E_{\mathbf{Fin}}G$, then the Davis-Lück assembly map*

$$\mu^{DL}_{E^G} : H^G(E_{\mathbf{Fin}}G, E^G) \to H^G(*, E^G)$$

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annihilates flasques: $K\mathcal{X}^G_*([0,\infty)\times X)\cong 0$

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Example: X - complete Riemannian spin proper G-manifold
have coarse index of Dirac operator (J. Roe):

$$\mathrm{ind}\mathcal{X}(D) \in K\mathcal{X}^G_{\mathsf{dim}(X)}(X)$$

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$p : X_{d,\mathcal{B}_{\mathrm{scal}}} \to *$ is proper and controlled

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with the following properties:

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introduce coefficients in a C^*-category $\mathbf{C}$

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for $f : X \to X'$ in $G\mathbf{BC}$

$$V^G_{\mathbf{C}}(f)(C, \rho, \mu) := (C, \rho, f_*\mu) \ , \quad V^G_{\mathbf{C}}(f)(A) := A$$

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excisive squares in $C^*\mathbf{Cat}^{\mathrm{nu}}$ are squares of the form

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preserves arbitrary products

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$(\mathbf{C}_i)_{i \in I}$ - family in $C^*\mathbf{Cat}^{\mathrm{nu}}$

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If $\mathbf{C}_i$ is additive for every i in I, then the canonical morphism

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Example: $A \to \mathbf{Hilb}_c(A)$ for unital A